

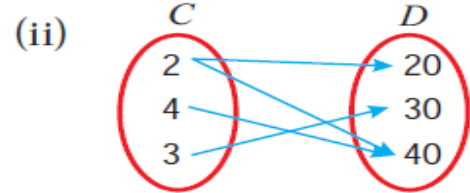
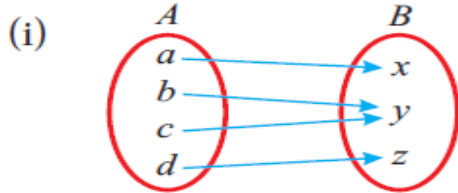
## 1. SETS AND FUNCTIONS

## Example problems

- For the given sets  $A = \{-10, 0, 1, 9, 2, 4, 5\}$  and  $B = \{-1, -2, 5, 6, 2, 3, 4\}$ , verify that (i) set union is commutative. Also verify it by using Venn diagram. (ii) Set intersection is commutative. Also verify it by using Venn diagram.
- Given,  $A = \{1, 2, 3, 4, 5\}$ ,  $B = \{3, 4, 5, 6\}$  and  $C = \{5, 6, 7, 8\}$ , show that  $A \cup (B \cap C) = (A \cup B) \cap C$  (ii) Verify (i) using Venn diagram.
- Let  $A = \{a, b, c, d\}$ ,  $B = \{a, c, e\}$  and  $C = \{a, e\}$ . (i) Show that  $A \cap (B \cap C) = (A \cap B) \cap C$ . (ii) Verify (i) using Venn diagram.
- Given  $A = \{a, b, c, d, e\}$ ,  $B = \{a, e, i, o, u\}$  and  $C = \{c, d, e, u\}$ . Show that  $A \setminus (B \setminus C) \neq (A \setminus B) \setminus C$ . (ii) Verify (i) using Venn diagram.
- Let  $A = \{0, 1, 2, 3, 4\}$ ,  $B = \{1, -2, 3, 4, 5, 6\}$  and  $C = \{2, 4, 6, 7\}$ . Show that  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ . (ii) Verify using Venn diagram
- For  $A = \{x / -3 \leq x < 4, x \in \mathbb{R}\}$ ,  $B = \{x / x < 5, x \in \mathbb{N}\}$  and  $C = \{-5, -3, -1, 0, 1, 3\}$ , Show that  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ .
- Use Venn diagrams to verify  $(A \cap B)^c = A^c \cup B^c$
- Use Venn diagrams to verify De Morgan's law for set difference  $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$ .
- Let  $U = \{2, -1, 0, 1, 2, 3, \dots, 10\}$ ,  $A = \{-2, 2, 3, 4, 5\}$  and  $B = \{1, 3, 5, 8, 9\}$ . Verify De Morgan's laws of complementation.
- Let  $A = \{a, b, c, d, e, f, g, x, y, z\}$ ,  $B = \{1, 2, c, d, e\}$  and  $C = \{d, e, f, g, 2, y\}$  Verify  $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$
- In a group of students, 65 play foot ball, 45 play hockey, 42 play cricket, 20 play foot ball and hockey, 25 play foot ball and cricket, 15 play hockey and cricket and 8 play all the three games. Find the number of students in the group.
- In a survey of university students, 64 had taken mathematics course, 94 had taken computer science course, 58 had taken physics course, 28 had taken mathematics and physics, 26 had taken mathematics and computer science, 22 had taken computer science and physics course, and 14 had taken all the three courses. Find the number of students who were surveyed. Find how many had taken one course only.
- A radio station surveyed 190 students to determine the types of music they liked. The survey revealed that 114 liked rock music, 50 liked folk music, and 41 liked classical music, 14 liked rock music and folk music, 15 liked rock music and classical music, 11 liked classical music and folk music. 5 liked all the three types of music. Find (i) how many did not like any of the 3 types? (ii) How many liked any two types only? (iii) How many liked folk music but not rock music?

14. Let  $A = \{1, 2, 3, 4\}$  and  $B = \{-1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12\}$ . Let  $R = \{(1, 3), (2, 6), (3, 10), (4, 9)\} \subseteq A \times B$  be a relation. Show that  $R$  is a function and find its domain, co-domain and the range of  $R$ .

15. Does each of the following arrow diagrams represent a function? Explain



16. Let  $X = \{1, 2, 3, 4\}$ . Examine whether each of the relations given below is a function from  $X$  to  $X$  or not. Explain. (i)  $f = \{(2, 3), (1, 4), (2, 1), (3, 2), (4, 4)\}$  (ii)  $g = \{(3, 1), (4, 2), (2, 1)\}$  (iii)  $h = \{(2, 1), (3, 4), (1, 4), (4, 3)\}$

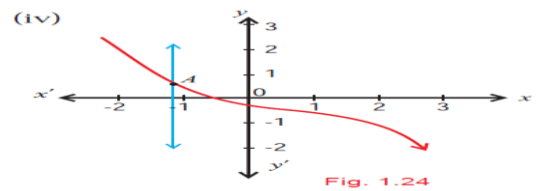
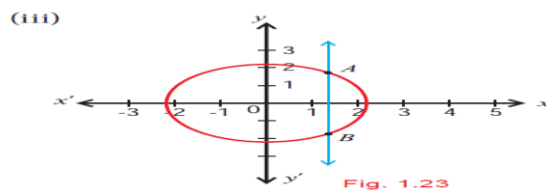
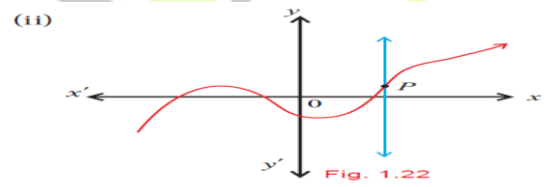
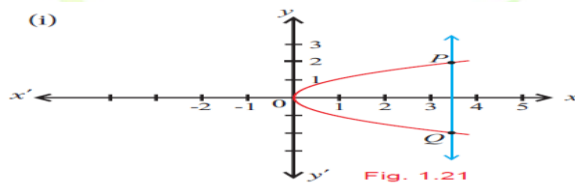
17. Which of the following relations are functions from  $A = \{1, 4, 9, 16\}$  to  $B = \{-1, 2, -3, -4, 5, 6\}$ ? In case of a function, write down its range.

(i)  $f_1 = \{(1, -1), (4, 2), (9, -3), (16, -4)\}$  (ii)  $f_2 = \{(1, -4), (1, -1), (9, -3), (16, 2)\}$

(iii)  $f_3 = \{(4, 2), (1, 2), (9, 2), (16, 2)\}$  (iv)  $f_4 = \{(1, 2), (4, 5), (9, -4), (16, 5)\}$

18. Let  $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$ , where  $x \in \mathbb{R}$ . Does the relation  $\{(x, y) \mid y = |x|, x \in \mathbb{R}\}$  define a function? Find its range.

19. Use the vertical line test to determine which of the following graphs represent a function.



20. If  $f: A \rightarrow B$  and  $f(x) = 2x + 1$ ,  $A = \{0, 1, 2, 3\}$ ,  $B = \{1, 3, 5, 7, 9\}$  represent  $f$  in

(i) Ordered pair (ii) Table (iii) Arrow diagram and (iv) Graph.

21. Let  $A = \{1, 2, 3, 4, 5\}$ ,  $B = \mathbb{N}$  and  $f: A \rightarrow B$  be defined by  $f(x) = x^2$ . Find the range of  $f$ . Identify the type of function.

22. Function  $f: [1, 6] \rightarrow \mathbb{R}$  is defined as follows  $f(x) = \begin{cases} 1 + x; & 1 \leq x < 2 \\ 2x - 1; & 2 \leq x < 4 \\ 3x^2 - 10; & 4 \leq x < 6 \end{cases}$

find the value of i)  $f(5)$  ii)  $f(3)$  iii)  $f(1)$  iv)  $f(2) - f(4)$  v)  $2f(5) - 3f(1)$