

## 7. TRIGONOMETRY

## Example problems

1. Prove the identity  $\frac{\sin \theta}{\csc \theta} + \frac{\cos \theta}{\sec \theta} = 1$
2. Prove the identity  $\sqrt{\frac{1-\cos \theta}{1+\cos \theta}} = \operatorname{cosec} \theta - \cot \theta$
3. Prove the identity  $[\operatorname{cosec}(90-\theta) - \sin(90-\theta)][\operatorname{cosec} \theta - \sin \theta][\tan \theta + \cot \theta] = 1$
4. Prove that  $\frac{\tan \theta + \sec \theta - 1}{\tan \theta - \sec \theta + 1} = \frac{1 + \sin \theta}{\cos \theta}$
5. Prove the identity  $\frac{\tan \theta}{1 - \cot \theta} + \frac{\cot \theta}{1 - \tan \theta} = 1 + \tan \theta + \cot \theta$
6. Prove the identity  $(\sin \theta + \csc \theta)^2 + (\cos \theta + \sec \theta)^2 = 7 + \tan^2 \theta + \cot^2 \theta$
7. Prove the identity  $(\sin^6 \theta + \cos^6 \theta) = 1 - 3 \sin^2 \theta \cos^2 \theta$
8. Prove the identity  $\frac{\sin \theta - 2 \sin 3 \theta}{2 \cos 3 \theta - \cos \theta} = \tan \theta$
9. Prove the identity  $\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = 1 - 2 \sec \theta \tan \theta + 2 \tan^2 \theta$
10. Prove the identity  $\frac{1 + \sec \theta}{\sec \theta} = \frac{\sin^2 \theta}{1 - \cos \theta}$
11. Prove the identity  $(\operatorname{cosec} \theta - \sin \theta)(\sec \theta - \operatorname{cosec} \theta) = 1 / \tan \theta + \cot \theta$
12. If  $\tan \theta + \sin \theta = m$ ,  $\tan \theta - \sin \theta = n$  and  $m \neq n$ , then show that  $m^2 - n^2 = 4\sqrt{mn}$ .
13. If  $\tan^2 \alpha = \cos^2 \beta - \sin^2 \beta$ , then prove that  $\cos^2 \alpha - \sin^2 \alpha = \tan^2 \beta$
14. A kite is flying with a string of length 200 m. If the thread makes an angle  $30^\circ$  with the ground, find the distance of the kite from the ground level. (Here, assume that the string is along a straight line)
15. A ladder leaning against a vertical wall makes an angle of  $60^\circ$  with the ground. The foot of the ladder is 3.5 m away from the wall. Find the length of the ladder.
16. Find the angular elevation (angle of elevation from the ground level) of the Sun when the length of the shadow of a 30 m long pole is 10.3 m.
17. The angle of elevation of the top of a tower as seen by an observer is  $30^\circ$ . The observer is at a distance of 30.3 m from the tower. If the eye level of the observer is 1.5 m above the ground level, then find the height of the tower.
18. A vertical tree is broken by the wind. The top of the tree touches the ground and makes an angle  $30^\circ$  with it. If the top of the tree touches the ground 30 m away from its foot, then find the actual height of the tree.

19. A jet fighter at a height of 3000 m from the ground passes directly over another jet fighter at an instance when their angles of elevation from the same observation point are  $60^\circ$  and  $45^\circ$  respectively. Find the distance of the first jet fighter from the second jet at that instant. (use  $\sqrt{3} = 1.732$ )
20. The angle of elevation of the top of a hill from the foot of a tower is  $60^\circ$  and the angle of elevation of the top of the tower from the foot of the hill is  $30^\circ$ . If the tower is 50 m high, then find the height of the hill.
21. A vertical wall and a tower are on the ground. As seen from the top of the tower, the angles of depression of the top and bottom of the wall are  $45^\circ$  and  $60^\circ$  respectively. Find the height of the wall if the height of the tower is 90 m. (use  $\sqrt{3} = 1.732$ )
22. A girl standing on a lighthouse built on a cliff near the seashore, observes two boats due East of the lighthouse. The angles of depression of the two boats are  $30^\circ$  and  $60^\circ$ . The distance between the boats is 300 m. Find the distance of the top of the lighthouse from the sea level.
23. A boy spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground level. The distance of his eye level from the ground is 1.2 m. The angle of elevation of the balloon from his eyes at an instant is  $60^\circ$ . After some time, from the same point of observation, the angle of elevation of the balloon reduces to  $30^\circ$ . Find the distance covered by the balloon during the interval.
24. A flag post stands on the top of a building. From a point on the ground, the angles of elevation of the top and bottom of the flag post are  $60^\circ$  and  $45^\circ$  respectively. If the height of the flag post is 10 m, find the height of the building. (use  $\sqrt{3} = 1.732$ )
25. A man on the deck of a ship, 14 m above the water level, observes that the angle of elevation of the top of a cliff is  $60^\circ$  and the angle of depression of the base of the cliff is  $30^\circ$ . Find the height of the cliff.
26. The angle of elevation of an aeroplane from a point A on the ground is  $60^\circ$ . After a flight of 15 seconds horizontally, the angle of elevation changes to  $30^\circ$ . If the aeroplane is flying at a speed of 200 m/s, then find the constant height at which the aeroplane is flying.